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Leon N. Cooper and James Rainwater

Nevis Cyclotron Laboratories
Columbia University
Physics Department
Irvington-on-Hudson
New York

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Theory of Multiple Coulomb Scattering from Extended Nuclei*

Leon N. Cooper and James Rainwater

Columbia University, New York, New York

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Uncertainties in the analysis of recent μ meson scattering experiments[1,2] have emphasized the necessity for a reasonably accurate estimate of the modification in the Coulomb multiple scattering distribution required to properly take into account the finite extension of the nucleus.

In particular, cosmic ray experiments[2] performed recently have been interpreted as indicating the existence of an anomalous (i.e. non-electromagnetic) μ-nuclear interaction which cannot be explained in terms of known μ meson interaction processes. In most of these experiments the multiple scattering distribution of relativistic μ mesons from 2 or 5 cm. lead plates is measured; then the experimental results are compared with the predictions of the Olbert[3] and Moliere[4] multiple scattering theories. Although by no means the only difficulty arising in the interpretation of such experiments, one of the most striking is the absence of a reliable estimate of the expected electromagnetic multiple scattering distribution at large angles.

In the Molière multiple scattering theory the nucleus is treated as a point charge. The single scattering cross section is taken to be the Rutherford cross section modified at small angles due to electron shielding. In the Olbert theory an attempt is made to estimate the effect of the nuclear extension by multiplying the single scattering law for projected angles by a step function which cuts off all single scattering beyond a certain projected angle. This gives a very great under-estimate of the multiple scattering at angles beyond the cut-off angle where the Olbert distribution has a Gaussian drop-off and soon falls greatly below any reasonable single scattering curve. It is easily seen that the correct multiple scattering curve should always fall above the single scattering curve at large angles (for a reasonable choice of the single scattering law).

We have therefore attempted to develop procedures for solving the multiple scattering problem starting with single scattering cross sections of the form

$$f(\varphi)\,d\varphi = \frac{Q}{2}\, F_N\,(\varphi/\varphi_0)\, \frac{d\varphi}{(\varphi^2+\varphi_m^2)^{3/2}}.$$

φ is the projected angle and φ_m is the screening angle. $F_N(\varphi/\varphi_c)$ is the nuclear form factor,[5] where $F_N(0) = 1$ and $F_N(\varphi/\varphi_c)$ decreases approximately as φ^{-4} for large values of φ. $\varphi_c = \frac{\hbar}{pR}$ and R is the nuclear radius.[6]

Two completely independent methods for solving this kind of a problem have been developed and applied to the case of the multiple scattering of relativistic μ mesons from 2 cm. of lead. The two methods give results that are in agreement with one another.

The first method might be characterized as a "brute force" numerical folding together of several partial distributions which together add up to the selected distribution of single scatterings. Such a method at first seemed hopelessly tedious, but several approximations were used which greatly speeded up the calculations without introducing excessive errors. The final calculation required about two days of slide rule and desk calculator time to obtain results which are estimated to be good to 1-2 percent at small angles and 5-10 percent at large angles (aside from errors in the assumed form factor). This method has the advantage of giving a good insight into the way the final result develops in terms of the physical processes.

The second method is an extension of the Molière theory in which $F_N(\varphi/\varphi_0)$ is included as part of the single scattering cross section. (The derivation will be given in an article to follow.) The multiple scattering law deduced by this second method can be expressed analytically and the difficult part of the calculation reduces to the evaluation of one

integral which can be done readily by numerical methods.

We obtain:

$$(a)\quad M(x) = \frac{e^{-x^2}}{\sqrt{\pi}} + \frac{1}{4G}\left\{ f'(x,\infty) - \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{d\xi\,(1-F_N(\xi/x_0))}{(\xi^2 + x_m^2)^{3/2}}\, T(x,\xi) \right\}$$

$$(b)\quad M(x) = \frac{e^{-x^2}}{\sqrt{\pi}}\left[1 + \frac{(2x^2-1)}{4G}\,q\right] + \frac{1}{4G}\frac{1}{\sqrt{\pi}} \int_K^\infty \frac{d\xi\, F_N(\xi/x_0)}{(\xi^2 + x_m^2)^{3/2}}\, T(x,\xi)$$

where $T(x,\xi) = \left[e^{-(x+\xi)^2} + e^{-(x-\xi)^2} - 2e^{-x^2}\right]$

and $q = 2\ln(K/1.26)$, where $0.1 \leq K \leq 0.5$ and

where x and xo are proportional to φ and φ_c.

Formula (a) is the same as Molière's formula except for the last term which gives the correction due to the nuclear extension. It is seen that if $F(\xi/x_0) = 1$ (point nucleus) the correction term is zero. For large values of x, formula (a) is inconvenient because the correction term is large compared to the net value of $M(x)$. In this case, however, (b) is convenient; especially so where x is large enough so that the first term in (b) can be neglected due to the smallness of e^{-x^2}. The two formulas can be used to evaluate $M(x)$ for all values of x. It is also to be noticed that a single calculation will serve for all momenta in the relativistic region as the integral depends upon $\beta \simeq 1$. The integrals were evaluated numerically using Weddle's[7] rule and grid spacings and values of K of 1/2 and 1/4. Comparison of the results for the two grid

spacings indicates that errors resulting from the numerical integrations are quite small.

We carried through calculations for the case (Figure 1) of 2 cm. Pb and cp = 1 Bev and used $F_N(\varphi/\varphi_0)$ = 1.00, 0.82, 0.50, 0.15 and $12(\varphi/\varphi_0)^{-4}$ for φ/φ_0 = 0, 1, 2, 3 and 4. This particular choice of F_N is intended to slightly underestimate the nuclear size effects, but otherwise to represent our best guess as to the "correct" form factor on the basis of recent experiments. We chose R = 1.1 $A^{1/3} \times 10^{-13}$ cm. and applied F_N to the law for projected scattering. This should give nearly the same result as choosing R = 1.0 x $A^{1/3} \times 10^{-13}$cm. with F_N applied to the law for total angle scattering. Inelastic scattering was not included here, but will be discussed in the article to follow.

REFERENCES

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(1) Whittemore, W. L. and Shutt, R. P., Phys. Rev. 88, 1312 (1952)

(2) George, E. P., Redding, J. L., and Trent, P. T., Proc. Phys. Soc. A, 66, 533 (1953)
Leontic, B. and Wolfendale, A. W., Phil. Mag. 44, 1101 (1953)

(3) Olbert S., Phys. Rev. 87, 319 (1952)

(4) Molière, G., Z. Naturforsch, 2a, 133 (1947);
3a, 78 (1948)

(5) Schiff, L. I., Phys. Rev. 92, 988 (1953)

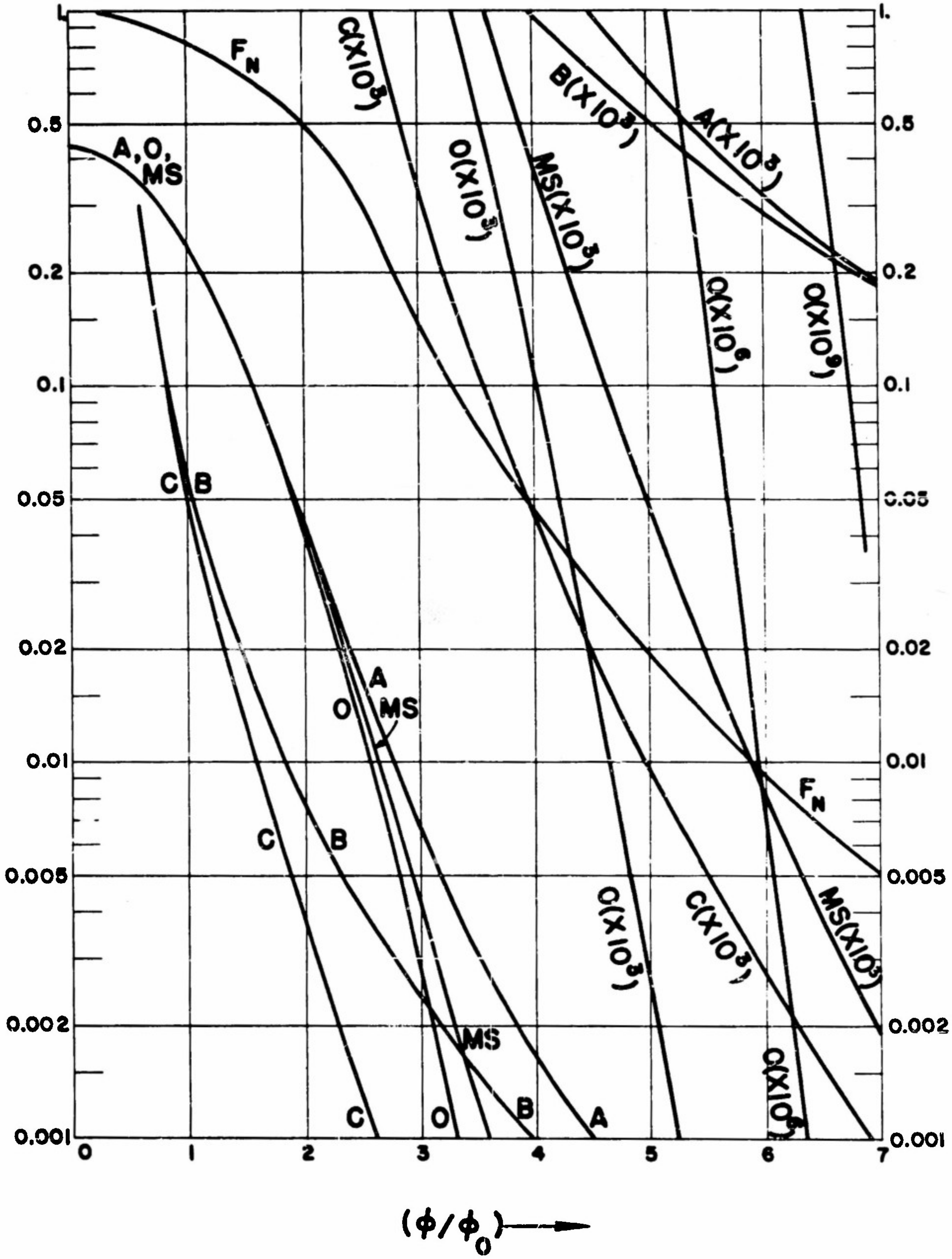
(6) See Olbert, S. (Ref. 3) for definitions of φ_m, x, G, Q, and other symbols used in the Modified Molière Theory.

(7) Morgenau and Murphy, The Mathematics of Chemistry and Physics, D. Van Nostrand, N. Y., (pg. 461)

R-81
CU-68

FIGURE CAPTION

Fig. 1 Curves appropriate to cp = 1 Bev and 2 c.m. Pb. Multiply (φ/φ_0) by 1.74 for Bev. degrees. Curve A = Molière; B = point nucleus single scattering; F_n = assumed nuclear form factor, C = single scattering law including F_n; MS = resulting multiple scattering law; O = Olbert distribution with single scattering cut-off at (φ/φ_0) = 1.1.



F_N
A, O, MS
C B
C(X10^3)
O(X10^3)
MS(X10^3)
B(X10^3)
A(X10^3)
O(X10^6)
O(X10^9)
A
O
MS
C
B
C(X10^3)
C(X10^3)
MS(X10^3)
O(X10^6)
C O B A
1.
0.5
0.2
0.1
0.05
0.02
0.01
0.005
0.002
0.001
0 1 2 3 4 5 6 7
(φ/φ0)